Problem D. Big Room

- For every wall cell, we want to determine the result of changing it to a floor cell.
  - The maximum of these results is the answer to the problem.
  - A naive approach would take $O(H^2W^2)$ time, which is too slow.
- Find the connected components of floor cells, and for each cell, determine:
  - The size of its connected component, $A[r][c]$.
  - The identifier of its connected component, $V[r][c]$.
- Only consider wall cells that are adjacent (up, down, left, or right) to at least one floor cell belonging to the connected component containing the start point.
- To calculate the result of changing a wall cell (r, c) to a floor cell:
  - Sum the $A[r][c']$ values of adjacent floor cells.
  - Ensure that $V[r][c']$ values are unique to avoid double-counting.

Problem E. Divide The Paper

1 Solution 1: Experimental Approach

- Plotting the results for various (H, W) pairs reveals a pattern.
- This pattern can be formalized into a mathematical condition.

2 Solution 2: Grundy Number Approach

- Vertical and horizontal cuts are independent operations.
- The game is equivalent to having two sticks of lengths H and W, where each turn a player splits one stick and discards the shorter piece.
- Compute Grundy numbers for the 1D version of the problem.
- The Grundy number sequence is conjectured as:

$$G[i] = \begin{cases} i/2 & \text{if } i \text{ is even,} \\ G[(i-1)/2] & \text{otherwise.} \end{cases}$$

- The winner is determined by the XOR of the Grundy numbers for H and W.

3 Solution 3: Mathematical Condition

- For $h \le w$, the losing positions satisfy $w = 2^k(h+1) - 1$ for some non-negative integer k.
- Example: For $h = 5$, the losing w values are $5, 11, 23, 47, 95, \ldots$.
- **Proof Sketch:**
  - If the initial position satisfies the condition:

    - For $h = w$ ($k = 0$), the second player can mirror moves to force a win.
    - For $h < w$ ($k > 0$), the second player can always restore the condition after any move by the first player.
  - If the initial position does not satisfy the condition, the first player can force the second player into a losing position.

Problem I. Attack Of Monsters

Using prefix sums, we can calculate the number of soldiers attacking the cell number i. Then we can just simulate the process. The total time is $O(n + m)$.