

## Problem A. Algebraically Universal Numbers

Input file:            `standard input`  
Output file:         `standard output`  
Time limit:          1 second  
Memory limit:       1024 megabytes

An integer  $a$  is called *algebraically universal* if there exists a real number  $b$  such that the equation  $a^x = b$  has infinitely many solutions in positive integers  $x$ .

You are given a number  $a$ , output 1 if it is algebraically universal, and 0 if it is not.

### Input

The input contains a single integer  $a$  ( $-1000 \leq a \leq 1000$ ).

### Output

Output 1 if  $a$  is algebraically universal, and 0 otherwise.

### Examples

standard input	standard output
2	0
-3	0

## Problem B. Conditions

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 256 mebibytes

You are given a list  $a$  that contains  $n$  different positive integers, a positive integer  $k$ , and a positive integer  $\ell$ . Find the smallest integer  $x$  that is greater than or equal to  $\ell$  and satisfies **exactly** one condition among the following ones:

- $x$  is in the list  $a$ ;
- $x$  is **not** divisible by  $k$ ;
- $x$  contains 7 in its decimal representation.

### Input

The first line contains two integers  $n$  and  $k$  ( $1 \leq n \leq 10^5$ ,  $2 \leq k \leq 10^5$ ).

The second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^{18}$ ).

The third line contains a single integer  $\ell$  ( $1 \leq \ell \leq 10^{18}$ ).

### Output

Output a single integer, the answer to the problem.

### Examples

<i>standard input</i>	<i>standard output</i>
5 9 181 182 184 178 81 173	183
1 888 888 888	888

### Note

In the sample, the integers 173, 174, 175, 176, 177, 179, 181, 182 satisfy two conditions. The integer 178 satisfies three conditions. The integer 180 does not fulfill any conditions. Finally, the integer 183 satisfies exactly one condition: it is **not** divisible by 9.

## Problem C. Modulo 4

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 256 mebibytes

Let  $A$  be the set of all arithmetic expressions consisting of the digits 0, 1, and the bitwise OR operator  $|$ , starting with 1, such that there is a 1 immediately after each  $|$ .

Let  $B_n$  be the subset of all expressions from  $A$  such that their value is equal to  $2^n - 1$  when considering the numbers in the expression in binary.

Let  $C_n$  be the subset of all expressions from  $A$  containing at least one expression from the set  $B_n$  as a suffix. For example, the following expressions are in  $C_3$ : 10011111, 111, 110|1|11, 11|11001|1010|101, and these expressions are not in  $C_3$ : 111|1011, 1, 10|11|11, 1100|10|100.

For given positive integers  $n$  and  $k$ , find the number of expressions from the set  $C_n$  that contain exactly  $k$  digits (and an arbitrary number of  $|$ ). As the answer may be very large, output it modulo 4.

### Input

The input contains  $t \leq 10$  test cases. The value  $t$  is given on the first line of input.

The only line in each test case contains two integers  $n$  and  $k$  ( $2 \leq n \leq 10^{12}$ ,  $n + 2 \leq k \leq 2 \cdot 10^{12}$ ).

### Output

For each test case, output the number of expressions from the set  $C_n$  containing exactly  $k$  digits, modulo 4.

### Example

<i>standard input</i>	<i>standard output</i>
4	2
2 4	0
5 15	1
147 10000	3
60 150	

### Note

Here are 14 expressions from the first example:

1111, 1011, 1|111, 111|1, 110|1, 11|11, 10|11, 11|10, 10|1|1, 11|1|1, 1|10|1, 1|11|1, 1|1|10, 1|1|11

## Problem D. Blind Gauss

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 256 mebibytes

Construct a square matrix with  $n$  rows and  $n$  columns consisting of nonnegative integers from 0 to  $10^{18}$  such that its determinant is equal to 1 and there are exactly  $a_i$  odd numbers in the  $i$ -th row for each  $i$  from 1 to  $n$ , or report that there is no such matrix.

### Input

The first line contains a single integer  $n$  ( $2 \leq n \leq 50$ ).

Each of the next  $n$  lines contains a single integer  $a_i$  ( $1 \leq a_i \leq n$ ).

### Output

If there is no matrix that meets the requirements, output -1.

Otherwise, output  $n$  lines with  $n$  numbers  $m_{i,j}$  in each ( $0 \leq m_{i,j} \leq 10^{18}$ ): the elements of the constructed matrix. If there are multiple solutions, print any one of them.

### Examples

<i>standard input</i>	<i>standard output</i>
2 1 1	1 0 0 1
2 2 1	1 1 1 2
4 3 3 3 3	1 0 1 1 1 1 1 2 1 1 2 3 0 1 1 3
3 2 2 2	-1
3 3 1 3	-1

## Problem E. Alice, Rooks, and Pawns

Input file:            `standard input`  
Output file:          `standard output`  
Time limit:           1 second  
Memory limit:        1024 megabytes

In Wonderland, there is a chessboard with  $n$  rows and  $m$  columns. Alice discovered that the Cheshire Cat placed  $r$  rooks on it, each controlling its own row and column. Alice needs to safely place  $k$  pawns on the board so that none of them is under attack from a rook. The chessboard in Wonderland is special: multiple pawns can be placed in the same cell at the same time!

Knowing all these rules, help Alice determine how many ways she can safely place all  $k$  pawns on the chessboard.

Two placements are considered different if there is at least one cell with a different number of pawns.

### Input

The first line contains four integers  $n$ ,  $m$ ,  $r$ , and  $k$  ( $1 \leq n, m \leq 2000$ ,  $1 \leq r \leq n \cdot m$ ,  $1 \leq k \leq 10^5$ ).

Each of the following  $T$  lines contains two integers  $h$  and  $v$  — the row and column number where the next rook is placed ( $1 \leq h \leq n$ ,  $1 \leq v \leq m$ ). No two rooks occupy the same square.

### Output

Output a single integer — the remainder of the number of different ways to place the pawns modulo 998 244 353.

### Example

standard input	standard output
3 2 1 3 1 1	4

## Problem F. Grand Prix of Array Count

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 256 mebibytes

An array  $a$  of length  $n$  is called *funny* if, for every pair of indices  $(i, j)$  where  $1 \leq i, j \leq n$ , the following condition holds: if  $i + j$  is an even number, then  $a_{(i+j)/2} = \gcd(a_i, a_j)$ . For example, an array  $[6, 2, 2, 2, 4]$  is funny.

You are given two positive integers  $n$  and  $k$ . Find the amount of funny arrays of length  $n$  consisting only of integers between 1 and  $k$ . As this number may be very large, output it modulo  $10^9 + 7$ .

### Input

The only line contains two integers  $n$  and  $k$  ( $5 \leq n \leq 10^{12}$ ,  $2 \leq k \leq 10^{12}$ ).

### Output

Print a single number: the answer to the problem modulo  $10^9 + 7$ .

### Examples

<i>standard input</i>	<i>standard output</i>
5 2	4
32 5	32

### Note

In the first sample, there are 4 funny arrays:  $[1, 1, 1, 1, 1]$ ,  $[1, 1, 1, 1, 2]$ ,  $[2, 1, 1, 1, 1]$ ,  $[2, 2, 2, 2, 2]$ .

## Problem G. Fans

Input file:            **standard input**  
Output file:         **standard output**  
Time limit:          1 second  
Memory limit:       1024 megabytes

The national football cup final will soon take place in Bytelandia. The two strongest teams in the country will face off: the capital's team "Double" and "Bytealona" from the city of the same name. Such matches are often called El Bytico. And if El Bytico happens to be the cup final, then all the fans will try to make it to the match...

Bytelandia can be represented as an infinite grid, where fans are located in various cells. The match will take place at cell  $(0, 0)$ , where the country's main stadium is located — the legendary The Infinity Stadium with infinite capacity.

At each step, fans can move in one of four directions: up, down, left, or right. However, there are  $n$  cells with obstacles, which contain no fans, and fans cannot pass through these cells. All fans who can reach the stadium in  $s$  steps or fewer will attend the match. Each fan will choose the shortest path to the stadium.

Fortunately or unfortunately, fan culture in Bytelandia is almost nonexistent: residents root "for beautiful football" and change the team they support at every step (i.e., before starting to move, each fan supports the Double club; after the first step — the Bytealona club; after the second step, they support Double again, and so on).

Write a program that determines how many fans who made it to the stadium will support the Double club, and how many will support the Bytealona club.

### Input

The first line of input contains two integers  $n$  and  $s$ : the number of obstacles and the maximum number of steps to reach the stadium ( $1 \leq n \leq 10^4$ ,  $1 \leq s \leq 10^7$ ). The next  $n$  lines each contain two integers representing the coordinates of obstacles. The absolute values of the coordinates do not exceed 1000. It is guaranteed that there is at most one obstacle per cell and that there are no obstacles at cell  $(0, 0)$ .

### Output

The first line of standard output should contain two integers separated by a space: the number of fans who arrived at the stadium supporting Double, and the number of fans who arrived at the stadium supporting Bytealona, respectively.

### Example

standard input	standard output
4 32768 2 2 -2 -2 2 -2 -2 2	1073807357 1073741824

## Problem H. Spruce and Oak Tiles

Input file:            `standard input`  
Output file:         `standard output`  
Time limit:          4 seconds  
Memory limit:       1024 megabytes

There are square wooden tiles of side 1, arranged in a rectangular grid with  $h$  rows and  $w$  columns. There are two types of tiles: spruce and oak. The type of tile in the  $r$ -th row from the top and  $c$ -th column from the left is denoted as  $s_{r,c}$ . Here,  $s_{r,c} = 1$  represents a spruce tile, and  $s_{r,c} = 0$  represents an oak tile.

You want to build a square frame using only spruce tiles. Your task is to determine the maximum possible side length of such a frame. A square frame of side length  $r_2 - r_1 + 1$  exists if there is a quadruple of integers  $(r_1, c_1, r_2, c_2)$  satisfying all the following conditions:

- $1 \leq r_1 \leq r_2 \leq h$
- $1 \leq c_1 \leq c_2 \leq w$
- $r_2 - r_1 = c_2 - c_1$
- For all  $(r_1, c)$  where  $c_1 \leq c \leq c_2$ ,  $s_{r_1,c} = 1$
- For all  $(r_2, c)$  where  $c_1 \leq c \leq c_2$ ,  $s_{r_2,c} = 1$
- For all  $(r, c_1)$  where  $r_1 \leq r \leq r_2$ ,  $s_{r,c_1} = 1$
- For all  $(r, c_2)$  where  $r_1 \leq r \leq r_2$ ,  $s_{r,c_2} = 1$

Write a program that, given the number of rows and columns of tiles and their types, determines the maximum possible side length of a square frame made of spruce tiles that can be constructed.

### Input

The first line of the input contains two integers  $h$  and  $w$  ( $1 \leq h, w \leq 2000$ ), representing the number of rows and columns of tiles, respectively. Each of the next  $h$  lines contains  $w$  integers  $s_{i,j}$  ( $0 \leq s_{i,j} \leq 1$ ), where  $s_{i,j} = 1$  denotes a spruce tile and  $s_{i,j} = 0$  denotes an oak tile.

### Output

Print the maximum possible side length of a square frame made of spruce tiles that can be constructed. If no such frame exists, print 0.

### Example

standard input	standard output
4 5 11110 10110 11011 11110	4



## Problem I. Try This at Home

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 256 mebibytes

For an integer array  $b$ , let's define  $f(b)$  as the lexicographically smallest array of the same length that is lexicographically greater than  $b$  and its **set** of elements is the same as  $b$ 's set of elements. If such an array does not exist,  $f(b)$  is undefined. For example,  $f([3, 2, 3, 2, 6, 6]) = [3, 2, 3, 3, 2, 6]$ . In this problem, "the set of array's elements" means an unordered collection of array's elements, where each element is only considered once regardless of how many times it appears in that array. Arrays  $[3, 2, 3, 2, 6, 6]$  and  $[3, 2, 3, 3, 2, 6]$  have the same set of elements  $\{2, 3, 6\}$ , despite some values appearing a different amount of times in the first and the second array.

Let  $f^k(b)$  denote the function  $f$  applied  $k$  times to  $b$ ; here, we consider  $f(\text{undefined})$  as undefined too. For example,  $f^2([3, 2, 3, 2, 6, 6]) = f([3, 2, 3, 3, 2, 6]) = [3, 2, 3, 3, 3, 6]$ .

You are given an integer array  $a$  of length  $n$ . In this problem, the array satisfies an additional constraint: **every** value in the array  $a$  appears there **at least twice**. Find the smallest positive integer  $k$  such that  $f^k(a)$  is not undefined and the array  $f^k(a)$  contains some value that appears exactly once in it, or report that there is no such  $k$ . As the answer may be very large, find it modulo  $10^9 + 7$ .

### Input

The input contains one or more test cases. The first line contains the number of test cases  $t$  ( $1 \leq t \leq 10^5$ ).

Each test case is given on two lines. The first of these lines contains an integer  $n$  ( $2 \leq n \leq 10^5$ ). The second line contains  $n$  integers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^6$ ). Every value in the array  $a$  appears there at least twice.

The sum of  $n$  for all test cases does not exceed  $6 \cdot 10^5$ .

### Output

For each test case, output a single line with the answer modulo  $10^9 + 7$ , or -1 if there is no answer.

### Example

<i>standard input</i>	<i>standard output</i>
3	1
6	-1
3 2 3 2 6 6	9
3	
6 6 6	
8	
1 1 4 3 3 4 1 1	

## Problem J. Pizza Restaurant

Input file: *standard input*  
Output file: *standard output*  
Time limit: 2 seconds  
Memory limit: 1024 mebibytes

You are given  $n$  strings  $s_1, s_2, \dots, s_n$ . Find any two **different** indices  $x$  and  $y$  and a **positive** integer  $k$  such that the string  $s_x \underbrace{s_y s_y \dots s_y}_{k \text{ times}}$  is a palindrome with length at most  $6 \cdot 10^6$ , or report that it is impossible.

### Input

The first line contains a single integer  $n$  ( $2 \leq n \leq 10^5$ ).

The next  $n$  lines contain  $n$  strings  $s_1, s_2, \dots, s_n$ , one per line ( $1 \leq |s_i| < 10^6$ ). The strings consist of lowercase English letters. The total length of all strings does not exceed  $10^6$ .

### Output

If there is no answer, output “No” (without quotes).

Otherwise, on the first line, print “Yes” (without quotes). On the second line, print three integers  $x, y$ , and  $k$  ( $1 \leq x, y \leq n$ ,  $x \neq y$ ,  $k \geq 1$ ,  $|s_x| + k \cdot |s_y| \leq 6 \cdot 10^6$ ). If there are multiple solutions, print any one of them.

### Examples

<i>standard input</i>	<i>standard output</i>
2 aa aa	Yes 1 2 1
4 a bb bcb cdc	No
2 ap papajoj	Yes 2 1 2

## Problem K. Unified Excursion Ticket

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           **3 seconds**  
Memory limit:        **1024 megabytes**

An old railway, operating as an open-air museum, can be represented as an array of  $n$  stations. When visiting the museum, a tourist buys a unified excursion ticket, which grants the right to an unlimited number of rides on the retro train.

Suppose the tourist is at some station. Then, they can take the retro train to any neighboring station (that is, from station  $i$ , if  $i$  is not 1 or  $n$ , they can go to stations  $i - 1$  and  $i + 1$ ).

Each station has a defined *impression* value  $a_i$ , which it makes on the tourist. If the tourist starts their journey from station  $i$ , their initial interest level  $x$  becomes equal to  $a_i$ . When visiting station  $j$ , the value of  $x$  is replaced by  $x \text{ AND } a_j$ . If the interest level  $x$  becomes zero, the tourist leaves the museum dissatisfied; otherwise, they visit all the stations in the museum and leave a positive review.

There are two types of events in the museum:

- Station  $i$  was renovated, and the value  $a_i$  for this station became  $y$ . This change remains in effect for subsequent events (until the station is renovated again, if that happens).
- A tourist arrived at the museum and decided to start their journey from station  $j$ .

For each event of the second type, determine the minimum number of moves between stations after which the tourist can leave the museum dissatisfied.

### Input

The first line of the input contains a single integer  $n$  — the number of stations on the museum railway ( $2 \leq n \leq 10^5$ ).

The second line contains  $n$  integers  $a_i$ , specifying the initial impression values for each station ( $0 \leq a_i < 2^{20}$ ).

The third line contains a single integer  $q$  — the number of events ( $2 \leq q \leq 10^5$ ).

Each of the following  $q$  lines describes one event. The events are listed in chronological order.

If renovation work occurred at station  $x$ , the event is given by three integers 1,  $x$ ,  $y$ , where  $x$  ( $1 \leq x \leq n$ ) is the station number and  $y$  ( $0 \leq y < 2^{20}$ ) is the new impression value for this station.

If a tourist started a tour from station  $x$ , the event is given by two integers 2,  $x$ , where  $1 \leq x \leq n$  is the station number.

### Output

For each query of the second type, output a single number. If for some choice of route the tourist's interest level can drop to zero, output the minimum number of rides after which this can happen. If a positive review is inevitable, output  $-1$ .

## Example

standard input	standard output
7	-1
11 9 8 11 9 8 8	2
13	5
1 4 10	5
2 4	3
1 7 8	1
1 2 1	6
2 1	0
2 7	
2 7	
2 5	
2 2	
1 2 8	
1 7 0	
2 1	
2 7	